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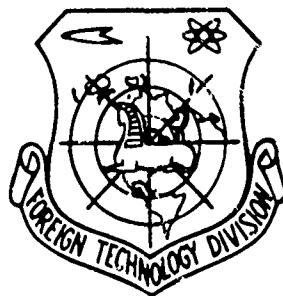
## FOREIGN TECHNOLOGY DIVISION



APPROXIMATION METHOD FOR THE CALCULATION OF  
TEMPERATURE IN A MICROCIRCUIT

by

F. A. Gur'yanova and S. A. Nikitin



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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
Б б	<i>Б б</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З з	<i>З з</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

\* ye initially, after vowels, and after ъ, ы; e elsewhere.  
 When written as ѣ in Russian, transliterate as я, е or ѣ.  
 The use of diacritical marks is preferred, but such marks  
 may be omitted when expediency dictates.

# APPROXIMATION METHOD FOR THE CALCULATION OF TEMPERATURE IN A MICROCIRCUIT

F. A. Gur'yanova and S. A. Nikitin

During the designing of integral microcircuits in a number of cases the knowledge of the average surface temperature of the support on which the components are arranged proves to be insufficient and knowledge of temperature at isolated points of its surface is necessary.

The computation of the temperature distribution on support, when on it large number of heat-releasing sources are arranged, represents a sufficiently complex and laborious problem which is usually solved with the use of a computer. However, a simple engineering procedure which makes it possible, rapidly and with satisfactory accuracy, to obtain information about the temperature at various points of a microcircuit is of definite interest.

A fine support (isotropic plate) is examined and the temperature differences in depth can be disregarded. This assumption is correct when the minimum size of the heat-releasing source is greater than the thickness of the support. The disregard of the temperature distribution in the depth of support for sources, one of dimensions of which is less than the thickness of the support,

gives rise to errors in determining the temperature on source, which apparently explains to some degree the divergence of results of calculation according to the proposed procedure and from experiment.

The support is found under conditions of free convection and in the range of temperatures in which microcircuits usually work it is possible to consider the heat-transfer coefficient independent of temperature. Its magnitude is calculated as the mean value of the heat-transfer coefficients from both sides of the support. The coefficient of thermal conductivity is taken independent from temperature. As a result of the thinness of support the convective heat removal from the ends can be disregarded.

For the indicated assumptions the stationary temperature distribution on the support can be described by the equation [2]

$$\frac{\partial^2 U(x, y)}{\partial x^2} + \frac{\partial^2 U(x, y)}{\partial y^2} - \alpha^2 U(x, y) = 0 \quad (1)$$

with the following boundary conditions:

$$\begin{aligned} \frac{\partial U(x, y)}{\partial n} \Big|_r &= 0, \\ -\lambda \frac{\partial U}{\partial n} 2(l_x + l_y) &= Q, \\ Q &= \oint_S \alpha U(S) dS, \end{aligned} \quad (2)$$

where  $U(x, y)$  - temperature gradient at a point with coordinates  $x, y$  with respect to the surroundings;

$\alpha$  - coefficient of heat transfer from the surface of the support;

$\lambda$  - coefficient of thermal conductivity;

$\delta$  - thickness of support;

$2(l_x + l_y)$  - the perimeter of the heat source;

Q - heat flux of the source;

$\Sigma$  - ends of support;

S - area of support;

$$\kappa^* = \frac{\alpha}{\lambda \delta}.$$

Using numerical methods on an M-20 computer S. A. Volkov obtained the solution of equation (1) with boundary conditions (2). The dimensions and coordinates of heat-releasing sources were arbitrary and were considered assigned, the thermal conductivity from the surface of source as a result of the smallness of the latter was not considered, and it was assumed that all the heat is drawn into the support. The approximate temperature distribution in a rectangular plate is given in work [1]. Specifically, if in the center of the support with sides  $L_x$  and  $L_y$  there is a heat-releasing source of a rectangular form, the sides of which are parallel to sides  $L_x$  and  $L_y$  and are equal respectively to  $l_x$  and  $l_y$ , then for temperature on the boundary of source the following expression is given

$$\Delta t = \frac{Q}{2\pi\lambda} \varphi(p, \gamma),$$

where Q - power of the source;

$$\varphi(p, \gamma) = \frac{1}{\pi p} \frac{l_1(\gamma) K_0(\gamma p) + K_1(\gamma) I_0(\gamma p)}{l_1(\gamma) K_1(\gamma p) - K_1(\gamma) I_1(\gamma p)}; \quad (3)$$

$$p = \frac{2l_n}{\sqrt{L_x^2 + L_y^2}}; \quad (4)$$

$$\gamma = \sqrt{\frac{2L_x L_y}{\pi \lambda \delta}} \sqrt{\frac{1 - \frac{l_x l_y}{L_x L_y}}{1 - p^2}}; \quad (5)$$
$$l_n = \min \{l_x, l_y\}.$$



Using the same principle - replacement of the real support of rectangular form by the equivalent plate of a circular form, as in G. N. Dul'nev's work [1], S. A. Volkov and Yu. A. Sher obtained the following expressions:

$$p = \frac{l_n}{\sqrt{L_x^2 + L_y^2}}, \quad (6)$$

$$\gamma = \sqrt{\frac{\pi L_x L_y}{\pi \lambda \delta}} \sqrt{\frac{\frac{\pi l_x l_y}{L_x + L_y}}{1 - p^2}}. \quad (7)$$

In the derivation of these relationships instead of the requirements for the equality of thermal conductivities  $\lambda$ , thickness  $\delta$ , and the perimeters of the heat-releasing sources of real and equivalent bodies the requirement was advanced for the equality of the areas of heat-releasing sources  $L_x L_y = \pi r_0^2$  and the average heat fluxes from the center of the source to the edges of the support

$$\lambda \frac{\theta_{\text{max}} - \theta_{\text{min}}}{\sqrt{(L_x/2)^2 + (L_y/2)^2}} = \lambda_0 \frac{\theta_{\text{max}} - \theta_{\text{min}}}{R},$$

where  $r_0$  - radius of the source of an equivalent plate;  
 $R$  - a radius of equivalent plate.

Under the condition of numerical agreement of average temperature indices of real and equivalent bodies, i.e., when

$$\theta_{\text{max}} - \theta_{\text{min}} = \theta_{\text{max}} - \theta_{\text{min}},$$

we obtain the expression

$$\frac{2\lambda}{\sqrt{L_x^2 + L_y^2}} = \frac{\lambda_0}{R}.$$

Table 1.

Values being calculated	Dimensions of support, cm.											$\frac{1}{\lambda} \cdot \frac{1}{cm}$
	$L_x = 4.8; L_y = 5.0$			$L_x = 2.4; L_y = 3.0$			$L_x = 2.0; L_y = 4.8$					
	Method of calculation											
	a	b	c	a	b	c	a	b	c			
$\rho$		0.045	0.023		0.031	0.015		0.025		0.012		
$\gamma$	1.37	1.68	1.59		0.84	0.75		0.97		0.89		
Error, %		32	14	1.82	1.75	2.22	1.71	1.6	13	1.94	0.0154	
$\gamma$	1.24	2.3	2.18		1.15	1.02		1.33		1.22		
Error, %		35	15	1.56	1.31	1.7	1.46	1.28	9	1.59	0.0288	
$\gamma$	1.12	3.25	3.41		1.63	1.45		1.87		1.72		
Error, %		39	22	1.36	1.05	1.37	1.26	1.05	6	1.33	0.0575	
$\gamma$	1.0	4.59	4.37		2.31	2.05		2.65		2.43		
Error, %		41	22	1.22	0.9	1.18	1.12	0.91	5	1.16	0.115	

\*The errors of computation were determined relative to data obtained on a computer.

Table 1 gives the dimensionless temperatures

$$\phi = \frac{\Delta t \lambda \delta}{Q},$$

where  $\Delta t$  - temperature gradient on the boundary of source relative to the surrounding medium.

The values of dimensionless temperature  $\phi$  are obtained from the solution of a problem of calculating the temperature on the boundary of a source arranged in the center of rectangular support by the following:

a) by means of solving on a computer the equation (1) with boundary conditions (2) for a source of rectangular form;

b) by means of computation of dimensionless temperature using formula (3) with parameters computed using formulas (4) and (5);

c) by means of computation of dimensionless temperature using formula (3) with parameters computed using formulas (6) and (7).

The mean value of the error of calculations for the variants, which are presented in Table 1, is 23% for method b and 12% for method c. Therefore in the subsequent calculations of  $\phi(\gamma, p)$  the computation of  $p$  and  $\gamma$  is made using formulas (6) and (7).

When on a support the source occupies any intermediate position between angular, lateral and central its temperature can be calculated with the help of dimensionless criterion  $N$ , determined by formulas [1]:

$$N = N_1 \operatorname{ch} \left( \frac{d_1}{d_{12}} \operatorname{Arch} \frac{N_1}{N_2} \right) \text{ when } N_2 > N_1, \quad (8)$$

$$N = N_2 \operatorname{ch} \left( \frac{d_2}{d_{12}} \operatorname{Arch} \frac{N_1}{N_2} \right) \text{ when } N_1 > N_2.$$

where  $d_1$  - the distance between sources with criteria  $N_1$  and  $N$ ;  
 $d_2$  - the distance between the sources with criteria  $N_2$  and  $N$ ;  
 $d_{12}$  - the distance between the sources with criteria  $N_1$  and  $N_2$ .

If the source is located in the center of the support, then  $N$  is connected with  $\varphi$  by the expression

$$N = \frac{1}{2} \varphi(\rho, \gamma). \quad (9)$$

Sources with criteria  $N_1$ ,  $N_2$ ,  $N$  should be arranged in such a way that their centers lie on one straight line. The criteria of sources located in the center, on the side, and in the corner, are connected by the relationship [1]

$$N_u : N_s : N_y = 1 : 2 : 4.$$

Substituting in (8) instead of criteria  $N_1$  and  $N_2$  the criterion for the central, lateral, and angular positions of a source, it is possible to calculate the family of curves which represents the location of sources which have an identical dimensionless criterion  $N$  (Fig. 1). An analogous problem was also solved on a computer, for which the source was placed in various places on the support (Fig. 2). It is evident from Fig. 1 that there is considerable difference in the results obtained using the approximation formulas (8) and from the exact solution of the problem on a computer. Therefore in the subsequent calculations the criteria  $N$  obtained on a computer will be used.

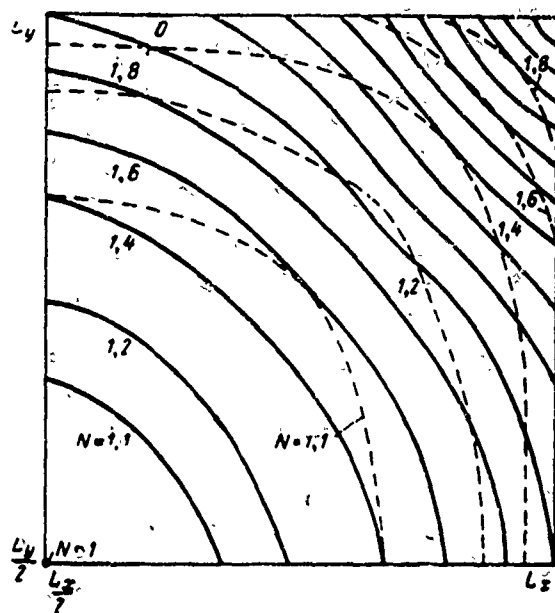


Fig. 1. Comparison of the results of the calculations of dimensionless criteria,  $N$  obtained using formula (8) (—) and on a computer (---).

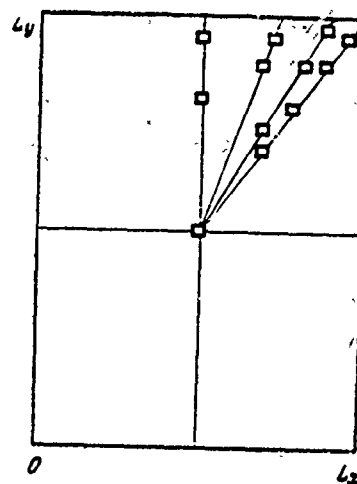


Fig. 2. The location of sources on a support during calculations on a computer.

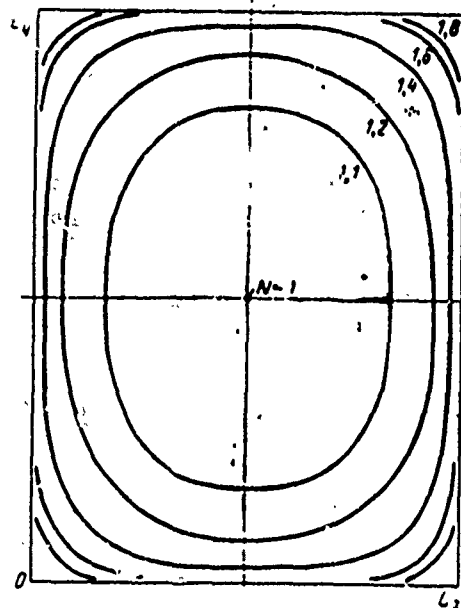


Fig. 3. The dependence of dimensionless criterion  $N$  on the location of the source on a support with the dimensions  $4.8 \times 6.0$  cm ( $M 1.25: 1$ ).

Figure 3 gives the curves which characterize the dependence of dimensionless criterion  $N$  on the location of the source on the support. The calculations for the determination of this dependence were performed for three dimensions of the support  $4.8 \times 6.0$  cm,  $2.4 \times 3.0$  cm and  $2.0 \times 4.8$  cm, and showed that dependences  $N(x, y)$ , constructed in relative coordinates, i.e.,  $N\left(\frac{x}{L_x}, \frac{y}{L_y}\right)$ ,

coincide for these dimensions of supports.

The graphs depicted in Fig. 3 make it possible to determine the temperature gradient on a source arranged in an arbitrary point with coordinates  $x$  and  $y$  on a support with dimensions  $L_x$  and  $L_y$ , referred to the temperature gradient on a source arranged in the center of a support. For this in Fig. 3 new coordinates are given for the heat-releasing source  $x'$ ,  $y'$  which are connected with the following assigned relationships:

$$x' = 1,25 \frac{4,8}{L_x} x, \quad y' = 1,25 \frac{6,0}{L_y} y, \quad (10)$$

where  $x$ ,  $y$ ,  $L_x$ ,  $L_y$  are expressed in centimeters and the value of dimensionless criterion  $N$  is determined. If necessary linear interpolation is conducted.

Computer calculations of the temperature field of a single source located in various places of a rectangular support showed that the isotherms represent ellipses which are distorted only in the vicinity of the source and at the edges of the support. The axes of the ellipses are parallel to the edges of the support and are related to each other approximately as the square of the corresponding sides of the support. Such a nature of isotherms in practice does not depend on the location of the source on the support. Consequently the simple scale conversion of support converts ellipse-isotherms into circumference-isotherms. This conversion makes it possible to present the temperature field of a single source in the form of the function of one coordinate  $\Delta r$  - the distance of the source on the converted support.

Figures 4 and 5 depict the dependence of the standardized temperature of support  $m$  in the function of distance from the source  $\Delta r$  for three directions - horizontal, vertical and diagonal, and for three positions of sources - central, lateral, and angular. Temperatures are standardized to temperature on the source.

Coordinates  $y$  in these figures are multiplied by the value  $(L_x/L_y)^2$ . From an examination of Figs. 4 and 5 it may be concluded that the form of the isotherms is close to circumferences.

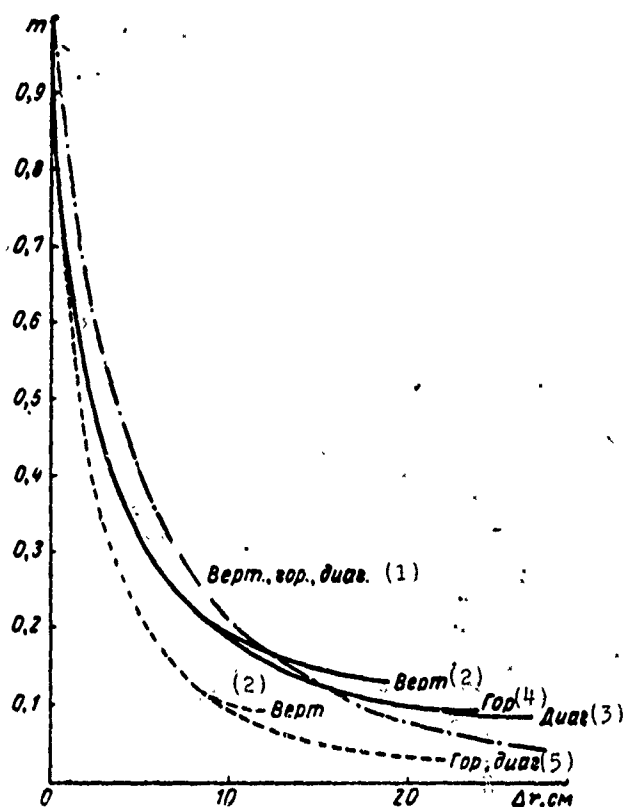


Fig. 4. The dependence of standardized temperature of supports on the distance  $\Delta r$  in a horizontal, vertical, and diagonal directions. The dimensions of the supports are  $4.8 \times 6.0$  cm and  $2.4 \times 3.0$  cm: — central; --- lateral; - - - angular location of source. KEY: (1) Vert., hor., diag.; (2) Vert.; (3) Diag.; (4) Hor.; (5) Hor., diag.

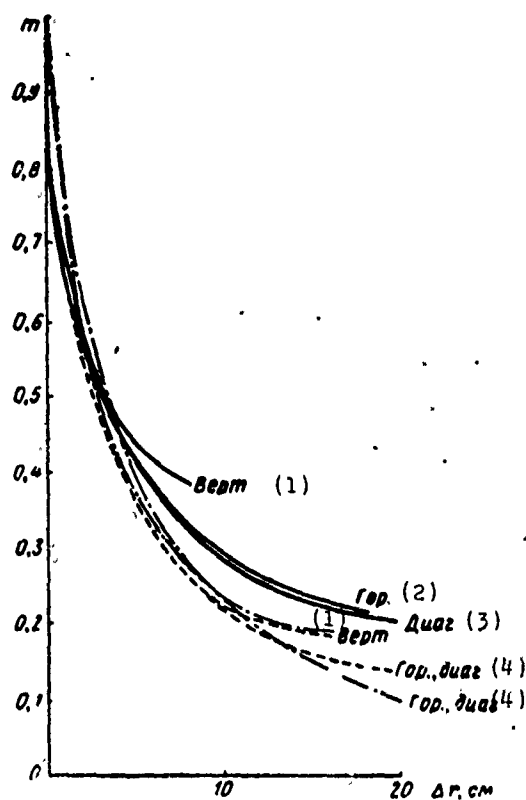


Fig. 5. The dependence of the standardized temperature of the support  $2.0 \times 4.8$  cm on the distance  $\Delta r$  in horizontal, vertical, and diagonal directions: — central; --- lateral; - - - angular location of source. KEY: (1) Vert.; (2) Hor.; (3) Diag.; (4) Hor., diag.

Figure 6 depicts the dependences of standardized temperature  $m$  on the distance  $\Delta r$  on a converted support with the dimensions  $4.8 \times 6.0$  cm at different values of the coefficient  $\kappa = \alpha/\lambda\delta$ .

Also written down in this figure are the temperature values calculated for other dimensions of supports and coefficients  $\kappa^2$ . It is easy to see that agreement of temperatures is observed when there is agreement of values  $\kappa^2(L_x^2 + L_y^2)$ , i.e., agreement of Biot's criteria.

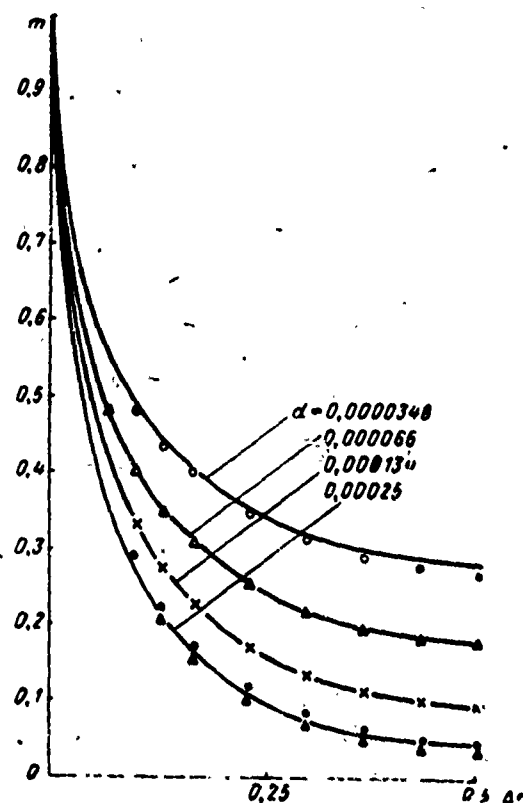


Fig. 6. The dependence of standardized temperature  $m$  on the distance  $\Delta r$  for a support with the dimensions of  $4.8 \times 6.0$  cm:  $\circ$  - dimensions of support  $2.4 \times 3.0$  cm,  $\alpha = 0.000134$ ;  $\Delta$  - dimensions of support  $2.4 \times 3.0$  cm,  $\alpha = 0.00025$ ;  $\bullet$  - dimensions of support  $1.6 \times 6.0$  cm,  $\alpha = 0.000382$ ;  $\blacktriangle$  - dimensions of support  $4.8 \times 4.8$  cm,  $\alpha = 0.000322$ ; here  $\alpha$  [cal/cm<sup>2</sup>·deg].

Figure 7 shows the family of dependences of standardized temperature on the distance from the source in a converted support for the various values of Biot's criterion. From Fig. 7 it is



possible to determine temperature at distance  $\Delta r$  from the source if the temperature on the source itself and the value of Biot's criterion  $\alpha/\lambda\delta(L_x^2 + L_y^2)$  are known.

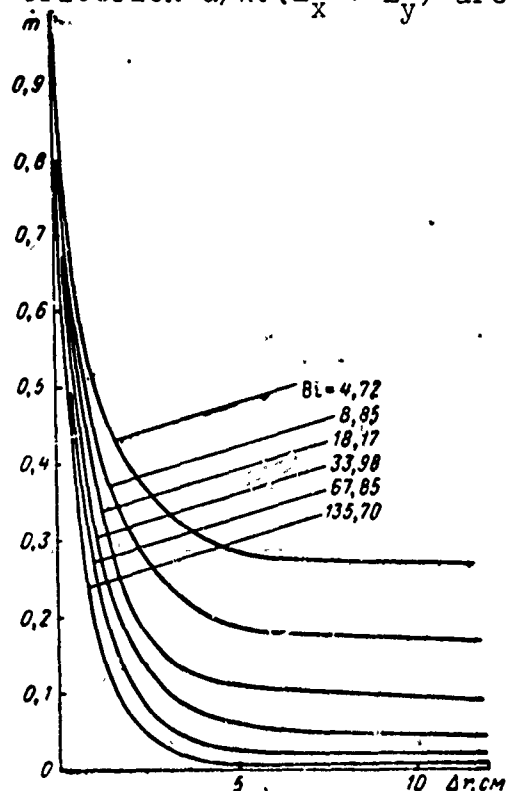


Fig. 7. The family of dependences of standardized temperature on the distance  $\Delta r$ .

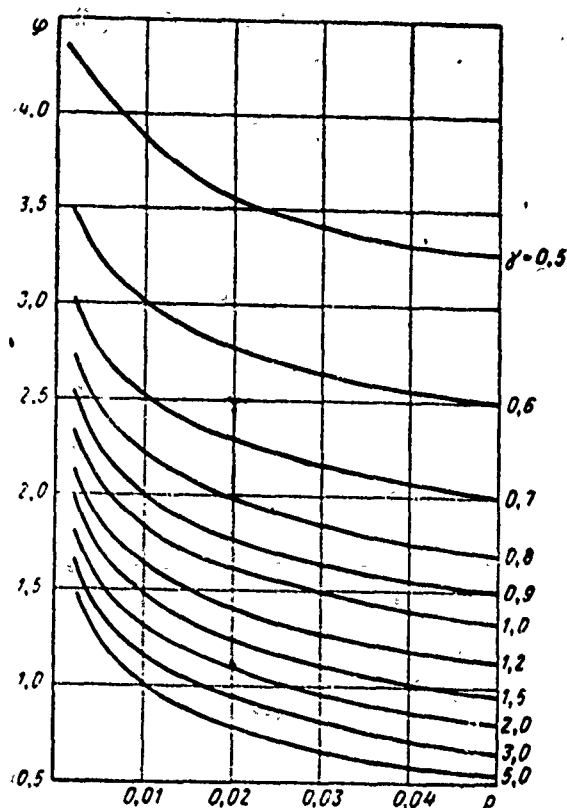


Fig. 8. Dependence of  $\varphi$  on  $p$  at different values of  $\gamma$ .

On the basis of what was expounded above it is possible to compile the following sequence for the approximate computation of temperatures in a microcircuit.

1. Using formulas (6) and (7) for every source  $\gamma_i$  and  $p_i$  ( $i$  - the number of source) are determined and using formula (3) or from the graph in Fig. 8 the values of dimensionless temperatures  $\varphi_i$  are found.

Using the formula

$$\Delta t_i = \frac{Q_i \varphi_i}{2\lambda\delta}$$

the value of temperature on every source is determined.

2. On tracing paper a rectangle is traced with sides 6 cm along the axis Ox and 7.5 cm along the axis Oy, in which the coordinates of heat-releasing sources are written as calculated using the formulas

$$x'_i = 6 \frac{x_i}{L_x} [\text{cm}],$$

$$y'_i = 7.5 \frac{y_i}{L_y} [\text{cm}],$$

where  $x_i, y_i$  - the true coordinates of heat-releasing sources;

$L_x, L_y$  - the actual sizes of the support. The figure obtained on tracing paper is superimposed on Fig. 3 and for every source the value  $N_i$  is found (if necessary linear interpolation is conducted).

For every source the value of  $\Delta t_i N_i$  is determined.

3. On a drawing grid a rectangle is constructed with the dimensions of 12 cm along the axis Ox and  $12 L_x/L_y$  cm along the axis Oy. In this rectangle the coordinates of sources are calculated using the formulas:

$$x''_i = 12 \frac{x_i}{L_x} [\text{cm}],$$

$$y''_i = 12 \frac{y_i}{L_y} \left( \frac{L_x}{L_y} \right)^2 [\text{cm}]$$

and on this figure the distances between the point at which the temperature is calculated (let us assume the point with coordinates  $x_j, y_j$ ) and the heat-releasing sources  $\Delta r_{ji}$  (in cm) are found.

We compute the value of Biot's criterion  $\alpha/\lambda\delta(L_x^2 + L_y^2)$ , we find from the graph (Fig. 7) that curve on which the dependence of the temperature on distance will be calculated, and we determine value  $m_{ji}(\Delta r_{ji})$ .

We compute the value of  $\Delta t_i N_i m_{ji} (\Delta r_{ij})$ .

4. The temperature difference in the point which interests us is found by using the formula

$$\Delta t_j = \sum_{i=1}^k \Delta t_i N_i m_{ji} (\Delta r_{ji}),$$

where  $k$  - the number of heat-releasing sources.

The value of the temperature is defined as

$$t_j = t_c + \Delta t_j,$$

where  $t_c$  - the temperature of the surrounding medium.

Table 2 gives the results of the calculation of temperature at the points of location of 14 sources by the expounded procedure and a comparison is made with the results of calculation on a computer.

Table 2.

$t_i$ in solution on computer, °C	50	52	54	56	56	56	58	57	57	57	54	48.5	49.5
$t_i$ in approxi- mate solution, °C	52	52	54	57	56	56	56	56	58	57	60	50	51

Table 3 gives the results of the calculations of temperature of a microcircuit with six heat-releasing sources which were obtained on a computer and by an approximate procedure, and their comparison with the results of an experiment (temperatures were calculated at the points of the location of the sources).

Table 3

$t_1$ in solution on a computer, °C	32	30	29	29	29	29
$t_1$ in an approximate solution °C	35.0	30.0	26.8	23.5	23.1	23.6
Experimental value $t_1$ , °C	29.3	27.8	27.4	27.2	27.2	27.2
Error of solution on a computer, %	9	8	6	7	7	7
Error of approximate solution, %	19	8	2	14	15	13

The calculations conducted show that the approximate procedure has an accuracy which is acceptable for engineer calculations and is distinguished by a high degree of simplicity and it can be recommended for calculations of the temperature fields of hybrid integral microcircuits with film resistors as heat-releasing sources.

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